

Research Article

Selection of MEMS Accelerometers for Tilt Measurements

Sergiusz Łuczak,¹ Robert Grepl,² and Maciej Bodnicki¹

¹*Institute of Micromechanics and Photonics, Faculty of Mechatronics, Warsaw University of Technology,
ul. Boboli 8, 02-525 Warsaw, Poland*

²*Institute of Solid Mechanics, Mechatronics and Biomechanics, Faculty of Mechanical Engineering, Brno University of Technology,
Technická 2896/2, 616 69 Brno, Czech Republic*

Correspondence should be addressed to Sergiusz Łuczak; s.luczak@mchtr.pw.edu.pl

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In order to build a tilt sensor having a desired sensitivity and measuring range, one should select an appropriate type, orientation, and initial position of an accelerometer. Various cases of tilt measurements are considered: determining exclusively pitch, axial tilt, or both pitch and roll, where Cartesian components of the gravity acceleration are measured by means of low-g uni-, bi-, tri-, or multiaxial micromachined accelerometers. 15 different orientations of such accelerometers are distinguished (each illustrated with respective graphics) and related to the relevant mathematical formulas. Results of the performed experimental study revealed inherent misalignments of the sensitive axes of micromachined accelerometers as large as 1°. Some of the proposed orientations make it possible to avoid a necessity of using the most misaligned pairs of the sensitive axes; some increase the accuracy of tilt measurements by activating all the sensitive axes or reducing the effects of anisotropic properties of micromachined triaxial accelerometers; other orientations make it possible to reduce a necessary number of the sensitive axes at full measurement range. An increase of accuracy while using multiaxial accelerometers is discussed. Practical guidelines for an optimal selection of a particular micromachined accelerometer for a specific case of tilt measurement are provided.

1. Introduction

Micromachined accelerometers belonging to Microelectromechanical Systems (MEMS) are employed in many kinds of various devices [1], owing to their small dimensions (currently even less than 2 mm), easy integration with electronics, high shock-resistance, high reliability, satisfactory accuracy, low power consumption, and low cost. They realize measurements of various types of acceleration (whether constant or variable), its derivatives (jerk, jounce) and integrals (velocity, position). New unexpected applications emerge constantly in, for example, smartwatches, underwater equipment like scuba diving computers (e.g., Vyper Air and Novo, Cobra 3, DX, D6i Novo by Suunto Company), new generation of safe motorcycle ABS [2] or traction control systems (e.g., Motorcycle Stability Control by Bosch Corp. [3]), or novel designs of commonly known objects, like the electronic gaming die presented in [4]. One of the most strategic fields of application is various types of navigation systems, proposed, for example, in [5, 6].

Besides typical measurements of acceleration related to vibration or motion, another basic application of accelerometers is tilt measurements [7]. Such measurements are employed in many devices, usually for the purposes of positioning, aligning, leveling, or navigating. However, these measurements are correct only under static or quasi-static conditions. Moreover, in the case of quasi-static conditions, especially at considerable frequencies, additional errors due to amplitude attenuation [8] must be taken into account.

As far as specific applications of tilt measurement are concerned, accelerometers are embedded in such equipment as various portable digital devices (e.g., cellular phones, photo cameras, computer projectors, GPS receivers, and game consoles). They can be applied directly in small devices like mobile microrobots [9] (especially when they are a part of the only navigation system available, like in the case of some in-pipe or biomimetic robots, proposed, e.g., in [10, 11]), or indirectly, as embedded in smartphones, as reported, e.g., in [12]. One can also find a lot of untypical applications of MEMS accelerometers used for tilt measurements related

to life sciences, for example, investigation of movements of animals [13] or humans [14–16] and rehabilitation equipment, for example, exoskeletons presented in [17–19], to name only a few.

Due to the fact that different mathematical formulas can be employed and different types of MEMS accelerometers are available, tilt measurements can be realized in various ways. Besides, orientation of the accelerometer is yet another important issue comprehensively addressed in the paper.

In order to perform a tilt measurement, the most advantageous solution is usually to apply a triaxial MEMS accelerometer and use arc tangent function [20]. Nevertheless, there are cases, specified in the paper, when other types of MEMS accelerometers are more preferable than triaxial sensors, for example, due to their inherent imperfections. Reasons for employing mathematical formulas other than arc tangent function are minutely discussed in [20]. For instance, if the accuracy of simple tilt measurements is not satisfactory, usually complex solutions are searched for, based, for example, on data fusion. Yet, sometimes a satisfactory increase of accuracy can be obtained by arranging an appropriate orientation of the MEMS accelerometer.

Despite common application of MEMS accelerometers, it is hard to find any satisfactory information on the aforementioned problems in the related publications. Therefore, we decided to address the issue of what type of MEMS accelerometer (uni-, bi-, tri-, or multiaxial) should be used in order to meet requirements related to certain case of tilt measurement, defining also its orientation and initial position as well as analyzing the respective sensitivity resulting from application of a given mathematical formula.

Owing to the guidelines presented in the paper, on the one hand, it is possible to build the simplest tilt measurement unit based on MEMS accelerometers, being at the same time the most cost-effective yet ensuring a satisfactory accuracy, and, on the other hand, to obtain possibly high accuracy at the cost of complicating the measurement.

2. Component Angles of Tilt

The most common way of expressing tilt is to determine its two component angles pitch α and roll γ (defined with accordance to aeronautical nomenclature, as, for example, in [21]) illustrated in Figure 1 against the components of the gravitational acceleration g . Often, it is the auxiliary angle β that is defined as the roll, for example, in [22–28], as in some cases such approach is more convenient, since accelerometers respond directly to this angle and not to angle γ .

Arbitrary tilt angle φ (axial tilt) included between acceleration g and axis z has been resolved into two component angles α (pitch) and γ (roll) or alternatively auxiliary angle β . Angles α and β are contained within vertical planes $x_0x_0z_0$ and kyz_0 , (the planes are generally not orthogonal with respect to one another), that is, angles between horizontal plane x_0y_0 and axis x or y , respectively. The coordinate system $x_0y_0z_0$ is immobile (its axis z_0 is vertical), whereas the coordinate system xyz is fixed to the mobile tilt sensor located at its origin. Angles α and β are interdependent and their values result from the value of angle φ . In order to illustrate β angle,

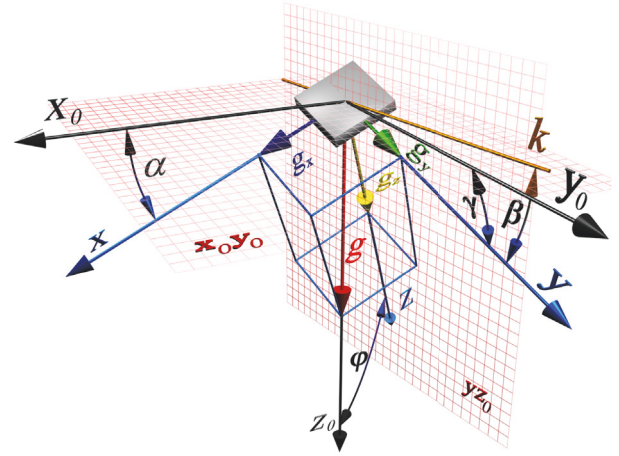


FIGURE 1: Components of the tilt and the gravitational acceleration.

line k has been created as a result of the intersection of vertical plane y_0yz and horizontal plane x_0y_0 .

Angle γ between axis y and y_0 is contained within a tilted plane y_0yz ; it is the angle by which the tilt sensor rotates around axis x , while tilted in two phases (the first phase is rotation by pitch α around axis y while it still overlaps axis y_0). It should be noted that most of the related formulas presented later in the text are analogous for angle α and β . Relation between the considered angles is as follows [21, 29]:

$$\gamma_{21} = \arcsin\left(\frac{\sin \beta}{\cos \alpha}\right), \quad (1)$$

$$\alpha = 0 \implies \gamma_{1i} = \beta_{1i}. \quad (2)$$

However, determining roll γ on the basis of (1) usually results in a decrease of the related accuracy [29] (see (21)).

From this point on, angles α , β , γ , and φ will be designated with additional subscripts, depending on the type of the considered tilt measurement (single- or dual-axis, as illustrated in Figure 2, or any of these two) and the formula employed for their determination, as specified in Nomenclature. The subscripts have been introduced in order to allow us to unequivocally distinguish between various cases introduced later in the text.

It was accepted that, in case of single-axis tilt measurements, the only component tilt angle that appears is pitch α .

3. Mathematical Formulas

The formulas presented in this section can be found in numerous related publications, for example, [20, 22–35]. As it results from Figure 1, the component tilt angles can be calculated basically as follows:

$$\alpha_{01} = \arcsin \frac{g_x}{g}, \quad (3)$$

$$\beta_{01} = \arcsin \frac{g_y}{g}. \quad (4)$$

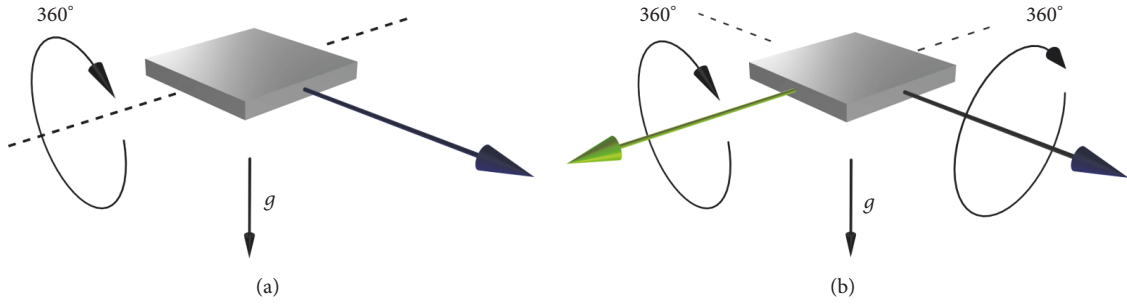


FIGURE 2: (a) Single-axis tilt sensor; (b) dual-axis tilt sensor.

However, the vertical component acceleration g_z allows us to use the following formulas:

$$\alpha_{22} = \arccos \frac{\sqrt{g_y^2 + g_z^2}}{g}, \quad (5)$$

$$\beta_{22} = \arccos \frac{\sqrt{g_x^2 + g_z^2}}{g}, \quad (6)$$

$$\alpha_{23} = \arctan \frac{g_x}{\sqrt{g_y^2 + g_z^2}}, \quad (7)$$

$$\beta_{23} = \arctan \frac{g_y}{\sqrt{g_x^2 + g_z^2}}, \quad (8)$$

and additionally,

$$\gamma_{03} = \arctan \frac{g_y}{g_z}. \quad (9)$$

If tilt is to be measured over one axis only, (5)–(8) get simplified, as one of the components of acceleration (g_x or g_y) will be then of zero value (e.g., (8) becomes identical to (9)). So, the equations will get transformed as follows:

$$\alpha_{12} = \arccos \frac{g_z}{g}, \quad (10)$$

$$\beta_{12} = \arccos \frac{g_z}{g}, \quad (11)$$

$$\alpha_{13} = \arctan \frac{g_x}{g_z}, \quad (12)$$

$$\beta_{13} = \arctan \frac{g_y}{g_z}. \quad (13)$$

While determining the arbitrary tilt angle φ , which in fact is an axial tilt defined in [33] and may be regarded as a pseudo dual-axis tilt measurement, the following relations are valid:

$$\varphi_{02} = \arccos \frac{g_z}{g}, \quad (14)$$

$$\varphi_{21} = \arcsin \frac{\sqrt{g_x^2 + g_y^2}}{g}, \quad (15)$$

$$\varphi_{23} = \arctan \frac{\sqrt{g_x^2 + g_y^2}}{g_z}. \quad (16)$$

If $g_x = 0$ or $g_y = 0$, then the measurement is actually a single-axis tilt measurement, where φ is determined instead of α .

Application of (14)–(16) may be interesting in such measurements of axial tilt as, for example, directional drilling [36] or while monitoring an object against losing its balance, since pitch and roll are inconvenient to be used in such cases.

Additionally, as discussed in Section 8, under certain conditions, measurement of axial tilt may be more advantageous than measurement of pure pitch, as far as the resultant accuracy is concerned.

Besides the presented formulas, there are still other ways of determining the tilt angles. As suggested in [24], it is advantageous in some cases to combine an arc sine formula ((3) or (4), (15)) with an arc cosine formula ((5) or (6) and (10) or (11), (14)) in a simple way or as a weighted average [30, 37].

4. Orientations of the Accelerometer

Possibilities of applying certain accelerometer in particular tilt measurement are illustrated in Table 1.

Formulas for determining pitch and roll given in Table 1 are listed for each case in the order of the highest measurement sensitivity (see Section 5); colors of the sensitive axes of the accelerometers, as they appear in the presented figures, agree with the accepted convention (see Nomenclature).

It should be added that obtaining a full measurement range of the component tilt angles is possible by observing the sign of the vertical component acceleration g_z , which is positive over the domain of $(-90^\circ; 90^\circ)$, while negative over the remaining angular range (thus, at least two sensitive axes of the accelerometer are required) [31].

Additional comments to the cases specified in Table 1:

- (i) Cases 1-A, 2-A, 2-B, 5-A, and 5-B are commonly applied and are well known.
- (ii) Case 1-B represents a pseudo dual-axis tilt (axial tilt) referred to in [33].

TABLE 1: Tilt sensors versus types of accelerometer.

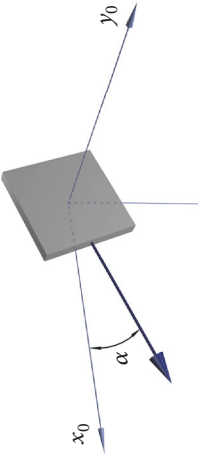
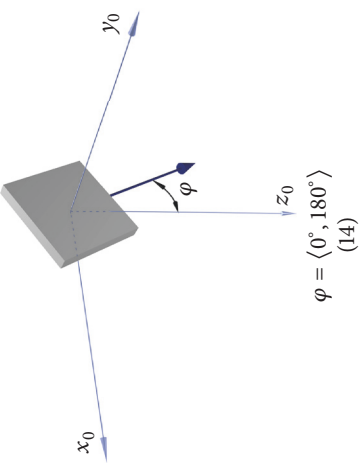
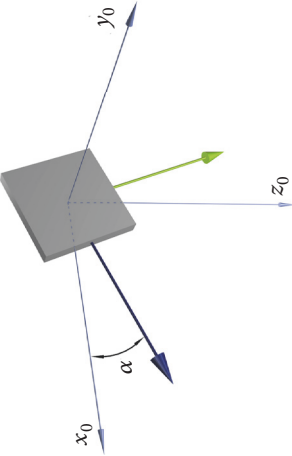
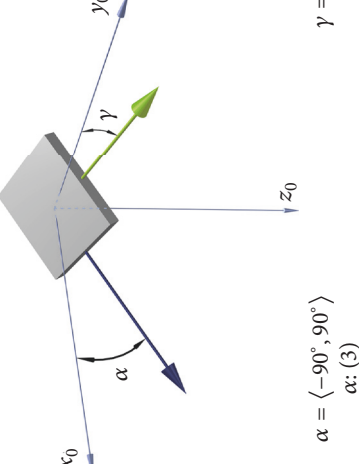
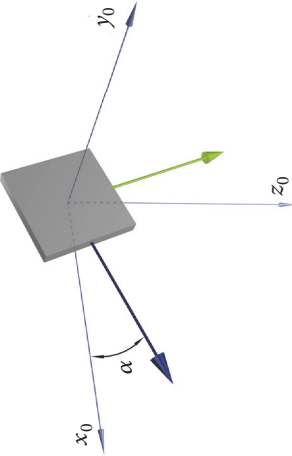
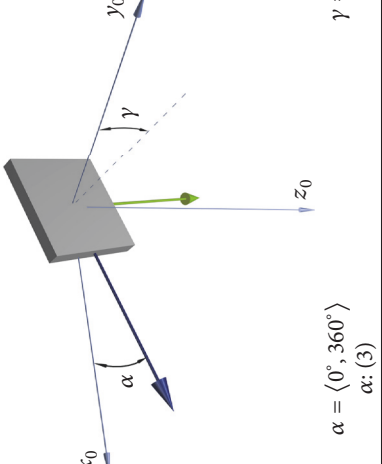
Number	Type of accelerometer	Type of tilt sensor	
1	Uniaxial	A	B
			
		Measurement range Mathematical formula	$\varphi = \langle 0^\circ, 180^\circ \rangle$ (14)
2	Biaxial		
		Measurement range Mathematical formula	$\alpha = \langle -90^\circ, 90^\circ \rangle$ $\alpha: (3)$ $\gamma = \langle -90^\circ, 90^\circ \rangle$ $\gamma: (4)^*$
3	Biaxial		
		Measurement range Mathematical formula	$\alpha = \langle 0^\circ, 360^\circ \rangle$ $\alpha: (3)$ $\gamma = \langle 0^\circ, 360^\circ \rangle$ $\gamma: (6)^*$

TABLE I: Continued.

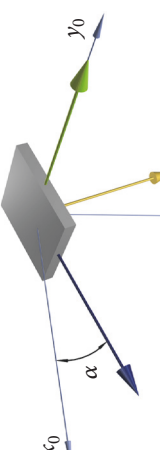
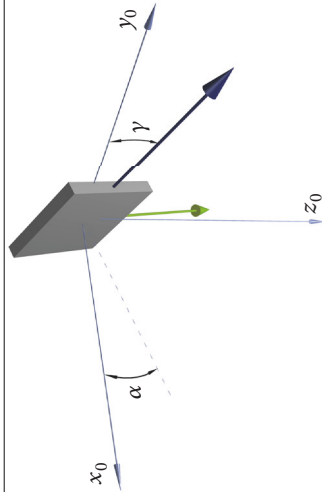
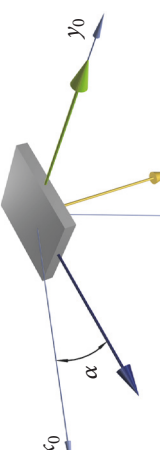
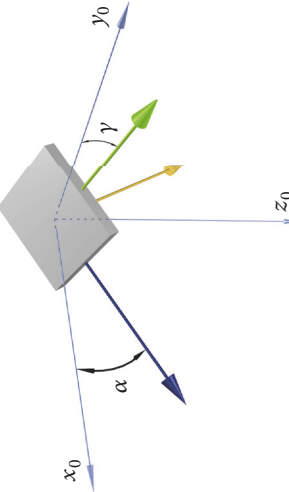
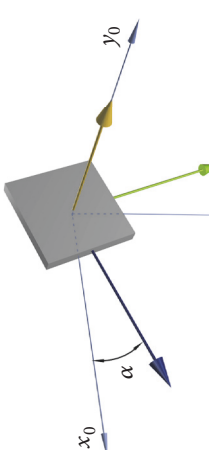
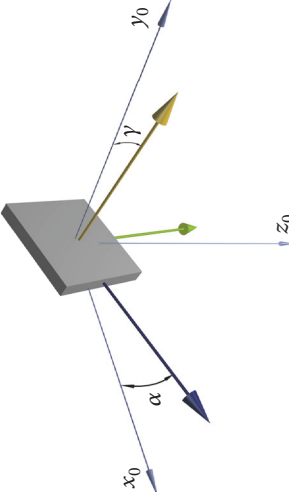
Number	Type of accelerometer	Type of tilt sensor	
4	Biaxial	A	B
			
		$\alpha = \langle 0^\circ, 360^\circ \rangle$ $\alpha: (5)$	$\gamma = \langle 0^\circ, 360^\circ \rangle$ $\gamma: (9); (4)^*$
5	Triaxial	A	B
			
		$\alpha = \langle 0^\circ, 360^\circ \rangle$ $\alpha: (12); (10); (3)$	$\gamma = \langle 0^\circ, 360^\circ \rangle$ $\gamma: (8)^*; (9); (6)^*; (4)^*$
		$\alpha = \langle 0^\circ, 360^\circ \rangle$ $\alpha: (7); (5); (3)$	$\gamma = \langle 0^\circ, 360^\circ \rangle$ $\gamma: (8)^*; (9); (6)^*; (4)^*$
6	Triaxial	A	B
			
		$\alpha = \langle 0^\circ, 360^\circ \rangle$ $\alpha: (12); (10); (3)$	$\gamma = \langle 0^\circ, 360^\circ \rangle$ $\gamma: (8)^*; (9); (6)^*; (4)^*$
		$\alpha = \langle 0^\circ, 360^\circ \rangle$ $\alpha: (7); (5); (3)$	$\gamma = \langle 0^\circ, 360^\circ \rangle$ $\gamma: (8)^*; (9); (6)^*; (4)^*$

TABLE 1: Continued.

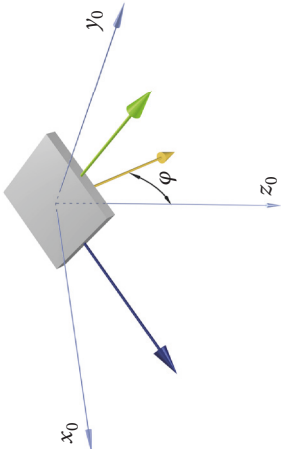
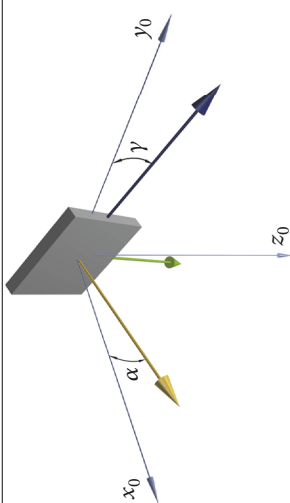
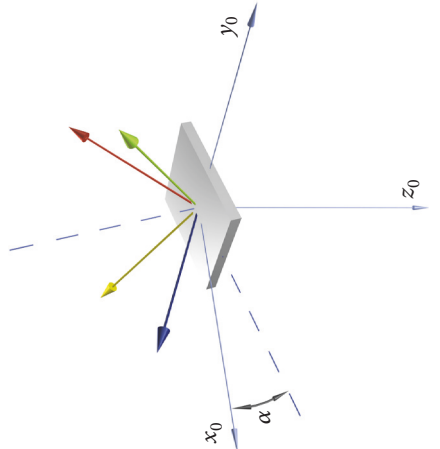
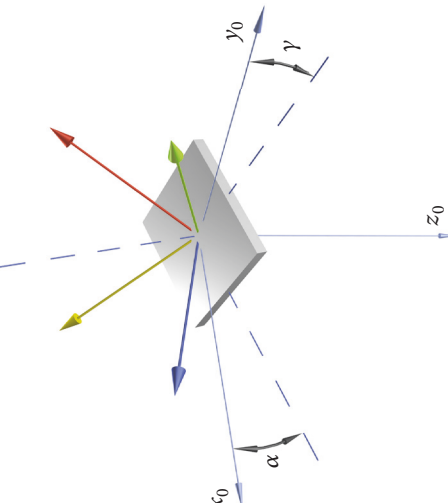
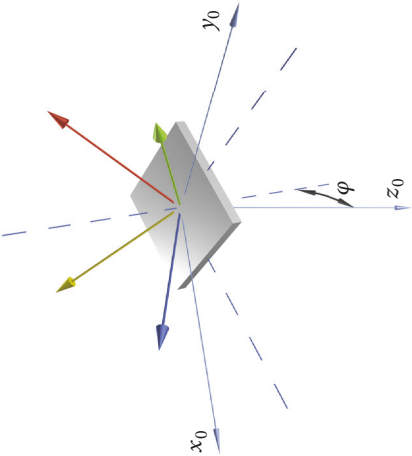
Type of tilt sensor	
Number	Type of accelerometer
7	A
	
	Triaxial
	B
	
	$\alpha = \langle 0^\circ, 360^\circ \rangle$ $\alpha: (7); (5); (3)$
	$\gamma = \langle 0^\circ, 360^\circ \rangle$ $\gamma: (8)^*; (9); (6)^*; (4)^*$
8	
	Multiaxial
	$\alpha = \langle 0^\circ, 360^\circ \rangle$ $(12)^{\dagger}; (10)^{\dagger}; (3)^{\dagger}$
	
	$\alpha = \langle 0^\circ, 360^\circ \rangle$ $\alpha: (7)^{\dagger}; (5)^{\dagger}; (3)^{\dagger}$
	$\gamma = \langle 0^\circ, 360^\circ \rangle$ $\gamma: (8)^{\dagger}; (9)^{\dagger}; (6)^{\dagger}; (4)^{\dagger}$

TABLE I: Continued.

Number	Type of accelerometer	Type of tilt sensor	
		A	B
9	Multiaxial		
	Measurement range	$\varphi = \langle 0^\circ, 360^\circ \rangle$	
	Mathematical formula	$(16)^\dagger; (15)^\dagger; (14)^\dagger$	

A: single-axis tilt sensor.
B: dual-axis tilt sensor.
* Roll γ must be determined using (1).
† Accelerometer indications must be first converted into Cartesian components (see (17)).
‡ Accelerometer indications must be first converted into Cartesian components (see (17)); then roll γ must be determined with (1).

TABLE 2: Initial positions of the accelerometers.

Number #	Case (Table 1)	Initial position of the accelerometer			
1	1-A	$s_1 \parallel x_0$			
2	1-A	$s_1 \parallel z_0$			
3	1-B	$s_1 \parallel z_0$			
4	1-B	$s_1 \perp z_0$			
5	2-A	$s_1 \parallel x_0$	$s_2 \parallel z_0$		
6	2-B	$s_1 \parallel x_0$	$s_2 \parallel y_0$		
7	3-B	$s_1 \parallel x_0$	$s_2 \parallel z_0$		
8	4-B	$s_1 \parallel y_0$	$s_2 \parallel z_0$		
9	5-A	$s_1 \parallel x_0$	$s_2 \parallel y_0$	$s_3 \parallel z_0$	
10	5-B	$s_1 \parallel x_0$	$s_2 \parallel y_0$	$s_3 \parallel z_0$	
11	6-A	$s_1 \parallel x_0$	$s_2 \parallel z_0$	$s_3 \parallel y_0$	
12	6-B	$s_1 \parallel x_0$	$s_2 \parallel z_0$	$s_3 \parallel y_0$	
13	7-A	$s_1 \nparallel x_0$	$s_2 \nparallel y_0$	$s_3 \parallel z_0$	
14	7-B	$s_1 \parallel z_0$	$s_2 \parallel y_0$	$s_3 \parallel x_0$	
15	8-A	$\angle s_1 x_0 = 35.3^\circ$	$\angle s_2 y_0 = 35.3^\circ$		
16	8-B	$\angle s_1 x_0 = 35.3^\circ$	$\angle s_2 y_0 = 35.3^\circ$		
17	9-A	$\angle s_1 z_0 = 54.7^\circ$	$\angle s_2 z_0 = 54.7^\circ$	$s_1 \nparallel x_0 z_0$	$s_2 \nparallel y_0 z_0$

s_1, s_2, s_3 , and s_4 : sensitive axes of the accelerometer.

#1–#4: arbitrary position of the accelerometer chip, typically like in 1-A, or 5-A.

#1, #3: when tilt angles $< 45^\circ$ are to be detected with higher sensitivity.

#2, #4: when tilt angles $> 45^\circ$ are to be detected with higher sensitivity.

#3–#4: additionally, expressions $s_1 \nparallel x_0 z_0$, $s_1 \nparallel y_0 z_0$ are true while tilted.

#10, #16: if $\alpha = 0$, then (9) and (13) are the same.

#15, #16: at the same time: $\angle s_3 y_0 = 144.7^\circ$, $\angle s_4 x_0 = 144.7^\circ$.

#17: at the same time: $\angle s_3 z_0 = 54.7^\circ$, $\angle s_4 z_0 = 54.7^\circ$, $s_3 \nparallel y_0 z_0$, and $s_4 \nparallel x_0 z_0$.

- (iii) Case 2-A, typically used for single-axis tilt measurement, can be accepted for dual-axis tilt measurement using an untypical orientation illustrated in cases 3-B and 4-B, which are an alternative to case 2-B; the resulting advantage is obtaining the full measurement range for both pitch and roll, using only 2 sensitive axes. Choice between 3-B and 4-B is related to the sensitivity of the measurements: at 3-B the highest sensitivity is obtained for $\alpha = 0^\circ$ and $\beta = 90^\circ$, whereas at 4-B for $\alpha = 90^\circ$ and $\beta = 0^\circ$; the sensitivity itself is variable, just as in case 2-B.
- (iv) Cases 5-A and 6-A are not recommended if it is striven for obtaining a higher sensitivity of the measurement, since none of the accelerometer sensitive axes should overlap axis y_0 while tilted, just as in case 7-A. However, additional calculations may be then necessary to convert angle φ into α . Analogously, the same applies accordingly to cases 8-A and 9-A.
- (v) For cases 6-A, 6-B, and 7-B, see Section 7 for explanation.
- (vi) Cases 8-A, 8-B, and 9-A refer to a sensor presented in [38], which has been used as an example of a multi-axial accelerometer. Application of such sensor makes it possible to increase the sensitivity of tilt measurements even by 13% [30].

While using a multi-axial accelerometer, before calculating tilt angles, Cartesian components of the gravitational

acceleration must be determined first. With regard to the accelerometer presented in cases 8-A, 8-B, and 9-A, the following formulas are true [38]:

$$\begin{aligned}
 g_x &= \frac{1}{2 \cdot \sin 54.7^\circ} (g_1 + g_4), \\
 g_y &= \frac{1}{2 \cdot \sin 54.7^\circ} (g_2 + g_3), \\
 g_z &= \frac{1}{4 \cdot \cos 54.7^\circ} (g_1 + g_2 + g_3 + g_4),
 \end{aligned} \tag{17}$$

where g_1, g_2, g_3 , and g_4 are projections of the vector of the gravitational acceleration onto the sensitive axes of the accelerometer, which are inclined by 54.7° (colors of the sensitive axes, represented by the bold arrows, denote their numbers with accordance to the accepted nomenclature).

Except for selecting an appropriate case of tilt measurement (as specified in Table 1), it is also important to set the initial position of the accelerometer in an advantageous way. The relevant data related to the cases illustrated in Table 1 are included in Table 2.

5. Sensitivity of Tilt Measurements

Even though the nominal values of the tilt angles calculated according to the presented equations are the same, yet they are determined with different sensitivity, thus different accuracy. So, application of particular formula significantly affects the accuracy of the measurement.

5.1. Sensitivity Related to Angles α , β , and φ . The appropriate formulas can be derived from (3)–(8), as demonstrated in [20]. They refer to pitch α in the same measure as to roll (while expressed by the auxiliary angle β), so a tilt angle ψ has been introduced. A relation between the sensitivity k_{01} (expressed in rad^{-1}) and the tilt angle ψ_{01} can be defined as [24]

$$k_{01} = g \cos \psi_{01}. \quad (18)$$

Likewise, for tilt angles ψ_{02} and ψ_{03} we can obtain [20]

$$\begin{aligned} k_{02} &= g \sin \psi_{02}, \\ k_{03} &= g = \text{const.} \end{aligned} \quad (19)$$

Sensitivities k_{01} and k_{02} , related to the arc sine and the arc cosine formulas, decrease down to zero at a tilt angle of 90° or 0° , respectively, what is very disadvantageous. However, application of the arc tangent formulas ((7)–(8), (12)–(13), and (16)) ensures a constant and at the same time the highest value of the sensitivity.

As far as the axial tilt φ is concerned, sensitivity k_{01} refers to φ_{02} , k_{02} refers to φ_{01} , and k_{03} refers to φ_{03} .

5.2. Sensitivity Related to Angle γ . On the basis of (1), the related sensitivity can be determined as

$$k_{21} = g \cos \alpha \sqrt{\frac{(\cos^2 \alpha - \sin^2 \beta)}{\sin^2 \alpha \cdot \sin^2 \beta + \cos^2 \alpha \cdot \cos^2 \beta}}. \quad (20)$$

The sensitivity k_{21} is expressed on the plot in g/rad ; thus, it is variable over the range of ca. $\langle 0; 10 \rangle \text{ m/s}^2 \text{ rad}$. It should be noted that not every combination of angles α and β is possible, since the angles are interdependent.

In the case of (9), the related sensitivity can be expressed by the following formula:

$$k_{23} = g \cos \alpha \quad (21)$$

which is also variable over the range of $\langle 0; 10 \rangle \text{ m/s}^2 \text{ rad}$, this time exactly as k_{01} . Both formulas are strongly nonlinear. However, sensitivity k_{23} is at least equal to k_{21} (often higher). Plot of (20) is illustrated in Figure 3.

In numerous applications, accuracy of tilt measurements (here represented by the sensitivity) is an important issue (even though there are some trivial cases when this problem is of no concern). Then, the most convenient mathematical relationship should be used, taking into account, first of all, the required accuracy and then simplicity of the related computations (thus, the resultant speed and cost of the measurement unit). Moreover, it must not be overlooked that the higher the accuracy of the measurements the higher the number of the sensitive axes of the accelerometer.

To conclude, it can be stated that in most of the cases the best solution is to use arc tangent formulas, that is, (7)–(9), (12)–(13), and (16), since they feature the highest sensitivity, provided that the number of the available sensitive axes is sufficient. However, there are also cases when the other formulas are preferred, as minutely discussed in [20].

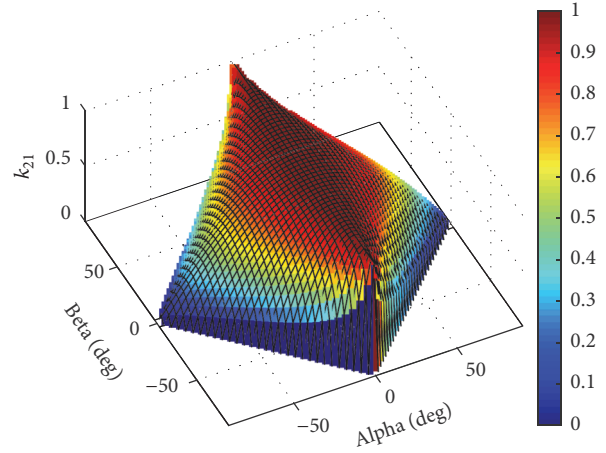


FIGURE 3: Variability of the sensitivity k_{21} expressed by (20).

It is worth mentioning that the sensitive axes need not to be orthogonal. Other spatial configurations are acceptable, and sometimes even more advantageous. Then, higher sensitivity can be obtained over some angular regions at the cost of its lower value over the other regions. Another disadvantage is the fact that the related computations are then more complicated, as briefly discussed in [30].

6. Accelerometer Selection

The required number of the sensitive axes results from type of tilt measurement, the measurement range, and the employed mathematical formula. Type of the selected accelerometer must comply with this number (the number of its sensitive axes may be alternatively higher).

While 3 sensitive axes are required, instead of employing 1 triaxial or 1 multiaxial accelerometer, it may be considered to apply 3 uniaxial or 2 biaxial accelerometers, or 1 uniaxial and 1 biaxial accelerometer. Such approach is more laborious and expensive; however it provides an opportunity to build a more accurate tilt sensor. This concept is accepted by such companies as MicroStrain Inc. or Sentera Technology Corp., whose sensors have the sensitive axes of the constituent accelerometers precisely calibrated and aligned (or have the relevant misalignments compensated for) [39].

However, even though some time ago triaxial MEMS accelerometers were not commercially available, nowadays it is quite the opposite: some manufacturers offer only triaxial devices, having abandoned production of biaxial or uniaxial versions. Yet, sometimes it is worthwhile to apply uniaxial or biaxial MEMS accelerometers. For instance, the alignment error of the sensitive axes of biaxial accelerometers is sometimes very small, that is, of 0.01° [28], unlike in the case of triaxial accelerometers, whose z -axis may be poorly aligned and additionally feature worse accuracy due to higher noise, what results from their mechanical structure determined by the fabrication process (this shortcoming is evident especially in the case of surface micromachining), which in fact is semi-three-dimensional [40].

TABLE 3: Characteristics of the considered mathematical formulas.

Equation	Sensitivity $\text{ms}^{-2}\text{rad}^{-1}$	Required number of sensitive axes
(3), (4)	10–0	1
(10), (11), (14)	0–10	1
(5), (6),	0–10	2
(1), (9), (15)	10–0	2
(12), (13)	10	2
(7), (8), (16)	10	3

Characteristics of particular mathematical formulas with respect to the sensitivity, the number of the required sensitive axes, and the measuring range of the applied accelerometers as well as complication of the related computation algorithm (proposed, e.g., in [31]) are minutely discussed in [20]. A brief summary is presented in Table 3.

One more problem to be considered is the operational parameters of the accelerometers. Two basic issues are the bandwidth and the supply voltage. If a possibly high accuracy is pursued, the supply voltage should be stabilized and of the highest value, whereas the bandwidth should be as small as possible, for example, 0–1 Hz. Then, the noise of the accelerometer is the lowest [28].

7. Imperfections of MEMS Accelerometers

Despite many aforementioned advantages of MEMS accelerometers, they feature some imperfections, of which the most significant, as far as tilt measurements are concerned, are thermal and long-term drifts of the output signals (both of the offset and the scale factor), misalignments (connected also with a lack of a physical datum surface and nonorthogonality of the sensitive axes), and cross-axis sensitivities. As far as thermal drifts are concerned, even some commercial sensors have this problem solved; for example, 16201 [41] of the ADIS series by Analog Devices Inc. Misalignments are dealt with rather seldom, for example, in the accelerometer described in [39]. However, referring to the cross-axis sensitivities, many models compensating for this errors have been proposed so far, for example, in [32, 36]. Less significant shortcomings are necessity of calibrating the accelerometer by the user (in the case of some accelerometers, custom equipment, like the one presented in [42], may be then required), anisotropy of the mechanical structure, sensitivity to electrostatic discharges, and thermal, mechanical, and electrical hysteresis.

However, if a basic type of accelerometer is used (with no compensation for the aforementioned shortcomings, yet much cheaper), and at the same time a relatively high accuracy of tilt measurements must be obtained, especially the thermal errors should be dealt with, as proposed, for example, in [22].

With regard to anisotropy, in some cases this imperfection can be eliminated. For instance, if a single triaxial accelerometer is used in a tilt measurement that does not require using all its output signals, the sensitive axis with the worst performance should be inactive. As a result from

the relevant catalog data, the signal related to vertical axis z of MEMS accelerometers (with respect to their packaging surface) is usually more noisy, compared to the horizontal axes x and y . So, it is more advantageous in single-axis tilt measurements to use the orientation of the accelerometer like in case 6-A (Table 1) instead of case 5-A.

As far as dual-axis measurements are concerned, two options are reasonable. If pitch α is to be determined with a higher sensitivity, orientation illustrated in case 6-B should be used. On the other hand, if it is roll γ that is more significant, the accelerometer should be oriented as in case 7-B.

With regard to the mechanical and electrical hysteresis it should be mentioned that its influence can be essentially reduced by subjecting the accelerometer to excited vibration (quasi-static conditions of operation). Moreover, as it results from our own experimental studies, this phenomenon is of minor significance in the case of MEMS accelerometers [43].

It must not be forgotten that each measurement case requires a precise alignment of the applied MEMS accelerometer, or appropriate compensation for the alignment errors, as addressed in [44]; otherwise a considerable loss of accuracy of determining the tilt must be taken into account.

As already mentioned, some of the listed imperfections are dealt with by manufacturers, who offer more sophisticated versions of the accelerometers, some even adjusted for tilt measurements, like ADIS 16201 by Analog Devices Inc. [41] or SCA100T by Murata Electronics Oy, called even an inclinometer [45].

8. Experimental Studies

In order to determine the expected increase of accuracy related to untypical orientations presented in Table 1, appropriate experimental studies were performed, in which we used the test rigs described in [42, 43] and two types of triaxial MEMS accelerometers ADXL 330 and 327 (two pieces of each type) by Analog Devices Inc.

First, the experiments revealed that the sensitive axes of the tested accelerometers were not perpendicular, with inherent mutual misalignments in the range of 0.05° – 1° (much higher than specified in the relevant datasheets). Moreover, values of the misalignments were different for each pair of axes and for each of the tested accelerometers. Such large value of the inherent misalignments of the sensitive axes of MEMS accelerometers is a very important issue in the case, when the misalignments are not compensated for in a numerical way (it is not possible to physically align the sensitive axes with respect to one another). As proven in [44], this kind of misalignment may directly increase the error of tilt measurement.

However, taking into account values of the inherent misalignments, it is possible to choose a more convenient orientation of the accelerometer, for example, case 6-A instead of 5-A, case 6-B or 7-B instead of 5-A. The result may be a decrease of the measurement error even as large as 1° , as our experiments proved.

By the way, it is not necessary to know values of the inherent misalignments. In order to choose a better orientation, a simple calibration should be performed for each orientation,

and on the basis of the obtained results, the more accurate orientation should be accepted.

Due to the aforementioned considerable inherent misalignments of the sensitive axes of the tested accelerometers it was not possible to evaluate accuracy increase related to single-axis measurements while accepting orientation specified in case 7-A as a better alternative to cases 5-A and 6-A (see Table 1). For the same reason, at this stage of experiments, it was not possible to verify how significant is the anisotropy of the tested triaxial MEMS accelerometers, and whether the related effects could be slightly reduced (applying case 6-A instead of 5-A; see Table 1).

Finally, in order to verify if it is more advantageous to use simplified formulas (10)–(13) instead of (5)–(8) in cases 5-A and 6-A, respectively, other series of experiments were performed. However, no significant difference was observed (relative changes of respective maximum errors as well as sums of errors were found to be no bigger than 0.4%). Nevertheless, in case of a bigger misalignment of the inactive sensitive axis (e.g., of ca. 1°), increase of accuracy may be considerable, as proved in [44].

Further experimental study will be performed in order to evaluate increase of accuracy of tilt measurement obtained owing to employing orientations specified in cases 6-A, 7-A, 6-B, and 7-B (see Table 1). Nevertheless, at this stage of experimental works (where the inherent misalignments have not been compensated for), it can be stated that the experimental results proved that it is worthwhile to reduce the inherent misalignments by choosing appropriate orientation of the accelerometer.

9. Conclusions

A practical approach to determining tilt with various types of MEMS accelerometers has been presented. The accepted idea of using components of the gravity acceleration for this purpose allows us to build a tilt sensor with miniature overall dimensions while employing MEMS accelerometers.

Results of our own experimental studies, reported, for example, in [24, 31], prove that using such accelerometers it is possible to detect tilt angles with accuracy of ca. few tenths of degree arc. However, our further experimental works reported in [42–44] revealed considerable imperfections of such accelerometers: the inherent misalignments between the sensitive axes (reaching up to 1°) as well as anisotropic metrological properties of the sensitive axes. Therefore, we proposed few original configurations of the applied MEMS accelerometer as one of the ways of reducing effects of the observed imperfections.

While arranging the discussed tilt sensor, the following issues must be considered (in the respective order):

- (i) Type of tilt measurement (single/dual-axis)
- (ii) Type of the applied accelerometers (uni-, bi-, tri-, or multiaxial; measurement range of the accelerometer should be ≥ 1 g, yet possibly small at the same time)
- (iii) Sensitivity of tilt measurement (determined mostly by the employed mathematical formula and noise of the accelerometer)

- (iv) Simplicity of mathematical processing of the output signals

Then, appropriate measurement case presented in Table 1 can be selected along with a corresponding initial position of the applied accelerometer, as specified in Table 2.

It must be realized that the best solution often does not exist. Usually, a trade-off must be sought for. Nevertheless, the paper shows how to build an optimal tilt sensing device. If typical arrangements, represented by cases 1-A, 2-A, 2-B, 5-A, and 5-B (see Table 1), do not satisfy certain requirements, other alternatives are proposed.

When a higher accuracy must be obtained, the following original orientations can be employed:

- (i) Cases 6-A, 6-B, and 7-B (see Table 1) make it possible to reduce the effects of anisotropic metrological properties of triaxial MEMS accelerometers; the accuracy can be increased even by 1° .
- (ii) cases 7-A and 9-A (see Table 1) make it possible to employ all the sensitive axes while realizing single-axis tilt measurements; the accuracy can be slightly increased; however, misalignments of the sensitive axes must be reduced to at least ca. 0.05° (whether physically or by compensation); otherwise, the accuracy may be even lower (as experimentally confirmed).
- (iii) A multiaxial accelerometer may be applied, as illustrated in cases 8-A, 8-B, and 9-A (see Table 1), to increase the accuracy (e.g., by 13%).

On the contrary, if for some reason (e.g., simplicity or speed of computations) the number of the sensitive axes should be reduced in a dual-axis tilt measurement over the full measurement range, cases 3-B and 4-B (see Table 1) may be accepted.

Still another issue is tilt measurement over a small range. At present, it is rather a difficult task using MEMS accelerometers, since only few models feature a small measuring range. Moreover, the range is relatively large anyway (e.g., 0.5 g in the case of SCA100T by Murata Electronics Oy, MXA6500EP or MXR2999EL by MEMSIC Inc.). However, it may be expected that smaller measuring ranges will be available soon, since some laboratory works in this field have already been started, as reported, for example, in [46].

It should be also mentioned that, in order to improve the accuracy, the output signals generated by MEMS accelerometers can be processed in a more sophisticated way, as proposed, for example, in [47]. Another way of achieving this goal is a sophisticated procedure of calibration of the accelerometers, as reported, for example, in [48, 49].

Nomenclature

g :	Gravitational acceleration
g_x, g_y, g_z :	Cartesian components of acceleration g
g_i :	Projection of acceleration g measured in i th sensitive axis of a multiaxial accelerometer

s_i :	i th sensitive axis of multiaxial accelerometer (1st, dark blue; 2nd, green; 3rd, yellow; 4th, red; observed in all the figures)
α :	Pitch
β :	Auxiliary tilt angle
γ :	Roll
φ :	Axial tilt (arbitrarily oriented tilt angle)
ψ :	Either pitch α or angle β
k :	Sensitivity
q :	Either α , β , γ , φ , ψ or k
q_{i1} :	Quantity “ q ” related to arc sine function
q_{i2} :	Quantity “ q ” related to arc cosine function
q_{i3} :	Quantity “ q ” related to arc tangent function
q_{0j} :	Quantity “ q ” related to either single- or dual-axis tilt measurement
q_{1j} :	Quantity “ q ” related to single-axis tilt measurement
q_{2j} :	Quantity “ q ” related to dual-axis tilt measurement
case X-Y:	Orientation defined in Table 1, Row X, Column Y.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors’ Contributions

Sergiusz Łuczak designed and wrote the manuscript; Robert Grepl analyzed sensitivity of determining roll angle and created the related graph; Maciej Bodnicki revised the manuscript; all the other figures were created by the acknowledged persons.

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